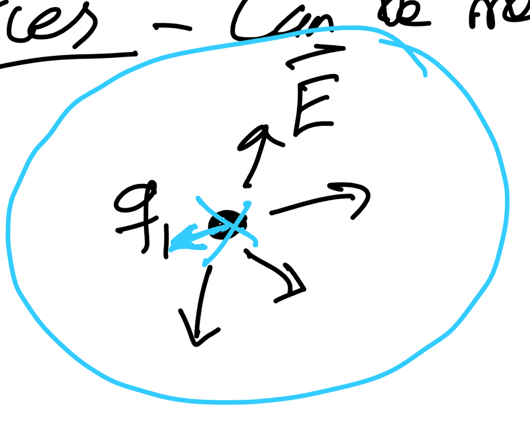
Lecture 1 Phys 313 Prof. Steven Anlage

Wednesday, August 26, 2026 2:55 PM

Electromagnetism = Electricity + Magnetism

Field Theory: Idealization of interactions between particles

Forces – Can be measured. $\vec{E}$



Test charge q_2 very very small

$\vec{F} = q_2 \vec{E}$ at charge q_2

measure

made up

Review of Mathematics:

scalar # with units

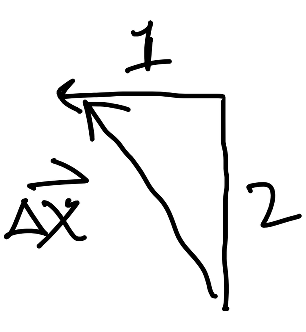
vector # magnitude + direction

$\frac{\rho}{\varepsilon_0}$ units

$\vec{\nabla} \cdot \vec{E} = \rho/\varepsilon_0$

Dimensional Consistency

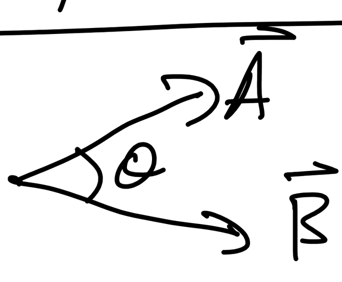
cartesian



$\vec{E}, \vec{B}$

$\vec{E}(x, y, z, t)$

Dot Product



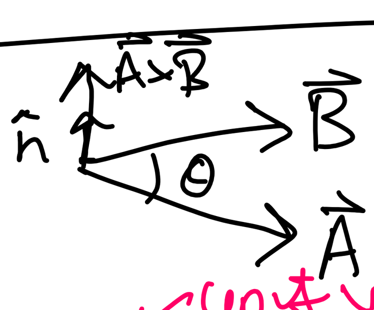
$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta$

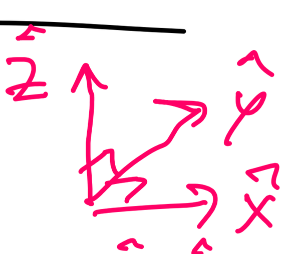
Dot product zero

QM Hilbert Space

$\langle \psi_1 | \psi_2 \rangle$ overlap

Cross product





$\hat{x} \cdot \hat{y} = 0$

$\hat{x} \cdot \hat{x} = 1$

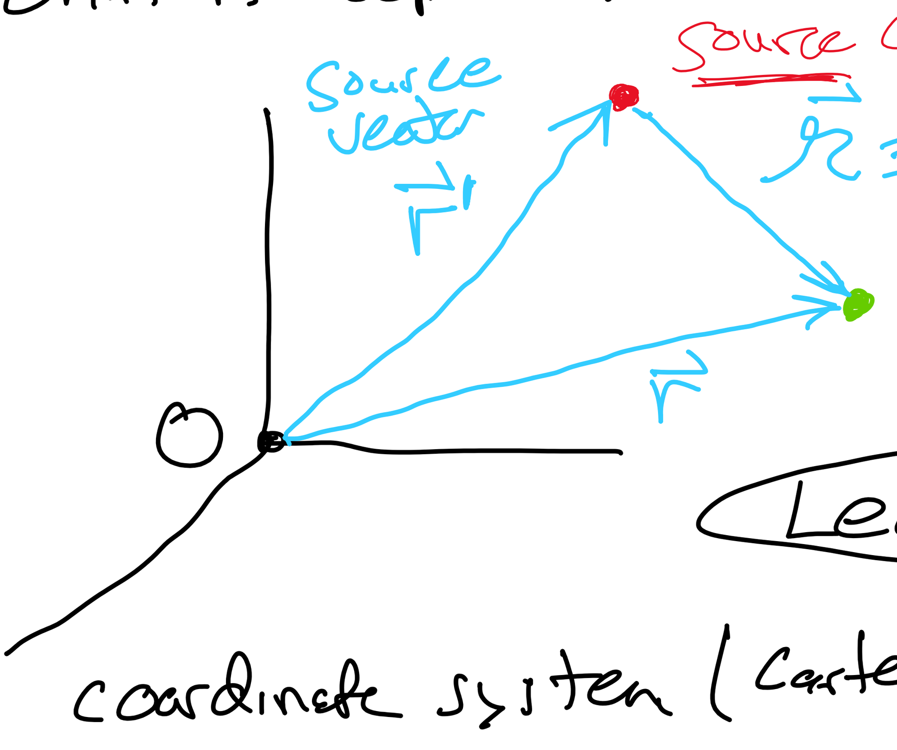
$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin\theta \, \hat{n}$

unit vector

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_y B_z - A_z B_y) - \hat{y}(A_x B_z - A_z B_x) + \hat{z}(A_x B_y - A_y B_x)$$

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}\underbrace{(\vec{A} \cdot \vec{C})}_{\text{scalar}} - \vec{C}\underbrace{(\vec{A} \cdot \vec{B})}_{\text{scalar}}$$

Griffiths Separation Vector



source charge (current, ...)

$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$ separation vector

Measure $\vec{E}$ field here

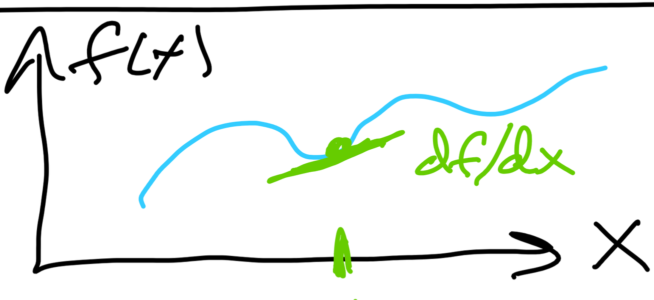
Field point $\vec{E}(\vec{r})$

Learn this

coordinate system (Cartesian, Cylindrical, Spherical)

☆ Origin

Differential Calculus



Generalize derivative

Gradient of a scalar function

$\rightarrow \vec{\nabla} h(x, y)$

Height of a hill

Direction: Steepest increase of $h(x,y)$ at same point.

Magnitude: Value of the greatest slope of the function at that point.

Gradient operator $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$ Vector operator

$\frac{\partial}{\partial x}\hat{x} = 0 \qquad \frac{\partial}{\partial x}\hat{y} = \frac{\partial}{\partial x}\hat{z} = 0$ unique to Cartesian

3-ways the gradient operator can act.

1. Bite a scalar function $\vec{\nabla} h$
2. Dot a vector function $\vec{\nabla} \cdot \vec{v}(x, y, z)$ Divergence
3. Cross product with a vector function $\vec{\nabla} \times \vec{v}(x, y, z)$ Curl.

The divergence

$\vec{\nabla} \cdot \vec{v}$

$\vec{v}(x, y, z) = v_x(x, y, z)\hat{x} + v_y(x, y, z)\hat{y} + v_z(x, y, z)\hat{z}$

3 scalar functions of x, y, z

$$\vec{\nabla} \cdot \vec{v} = \left(\cancel{\hat{x}}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \cdot \left(v_x \hat{x} + v_y \cancel{\hat{y}} + v_z \cancel{\hat{z}}\right)$$

$$= \frac{\partial v_x(x, y, z)}{\partial x} + \frac{\partial v_y(x, y, z)}{\partial y} + \frac{\partial v_z(x, y, z)}{\partial z}$$ Scalar

$\hat{x} \cdot \hat{y} = 0 \qquad \frac{\partial}{\partial x}\hat{y} = 0$

The Curl

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix} =$$

$$= \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) - \hat{y}\left(\frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

curliness of a vector field

Product Rules $\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$

scalar function

Second Derivatives Q = scalar function of x, y, z.

$\vec{\nabla} \cdot (\underbrace{\vec{\nabla} Q}_{\text{vector}})$ Divergence of a gradient

$\vec{v}$ = vector function of position

$\underbrace{\vec{\nabla} \times}_{\text{curl}} (\underbrace{\vec{\nabla} \times \vec{v}}_{\text{vector}})$

NOT WORKING $\cancel{\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{v})}$ scalar $\cancel{\vec{\nabla} \times (\vec{\nabla} \cdot \vec{v})}$ scalar

$\vec{\nabla} \cdot \vec{\nabla} = \nabla^2$ Laplacian operator

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$\vec{\nabla} \times (\vec{\nabla} Q) = 0$ Always

$\vec{\nabla} \cdot (\underbrace{\vec{\nabla} \times \vec{v}}_{\text{vector}}) = 0$ Always

$$\underbrace{\vec{\nabla} \times (\vec{\nabla} \times \vec{v})}_{\text{vector}} = \underbrace{\vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{v}}_{\text{scalar}})}_{\text{vector}} - \underbrace{\nabla^2 \vec{v}}_{\text{vector}}$$

Synthesis

$\underbrace{\vec{\nabla} \cdot \vec{E}}_{\text{scalar}} = \rho/\varepsilon_0$ scalar function $\rho = \rho(\cancel{x}, y, z)$

charge density $\frac{\#\text{Coulomb}}{m^3}$

Electric field diverges from charge.

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Resulting magnetic field

current density

scalars

vector

Sources of magnetic field